Impedance control of a planar quadrotor with an extended Kalman filter external forces estimator

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***Abstract*— In this work we deal with the non-linear control of aerial vehicles under external disturbances. We develop a non-linear velocity controller able to accommodate estimations of the external disturbing forces and moments. To estimate the external actions and at the same time provide improvements on the state estimation we make use of the EKF approach. Finally, we present simulations comparing close loop performance of a system with the proposed methodology implemented against close loop performance of the same controller but without the estimation of the external forces.**

I. Introduction

Recently, UAVs have evolved to cover successfully civilian tasks such as exploration, fire inspection in natural areas, agricultural inspection, aerial photography, aerial video recording, terrain mapping and many other tasks related with the environment sensing [1]. USA UAVs, for instance, provided real time imagery and video after the earthquake in Haiti in 2010 and the earthquake that led to a tsunami in Japan in 2011[1].

The high degree of effectiveness in UAV-applications [2], [3] besides the potential that UAVs have to solve civilian and military missions that are to come, is the main reason for the growing of research in the flight control systems field.

Nowadays, many studies are focused in enhancing the autonomy of unmanned vehicles. Consequently, tasks as autonomous guidance and trajectory tracking have been achieved successfully [4], [5]. Even more, some grasping tasks [6], [7], that evolve to autonomous construction [8], art inspired demonstrations [9], and cooperative work between UAVs [10] have been demonstrated on well structured environments.

Currently, UAVs are not only able to observe but also to interact with the environment. Recently finished and ongoing european projects are focused on control techniques that allow safe interaction between UAVs or between UAVs and the external world[2]. All of them have been motivated by the potential of this kind of aircraft as tools for damage assessment in critical areas after natural disasters or in hazardous environments, where walls and other physical barriers may exist.

In general when flying in unstructured environments it is very important to be immune to external actions or interaction forces that can not be predicted beforehand e.g. when the aircraft is in contact with external agents as loads, walls or is physically interconnected with other aircraft.

In this work we use the ideas of [11] and [12] to design a non-linear velocity controller for a simplified quadrotor platform, using only measurements from on-board sensors and an Extended Kalman Filter (EKF) for state and external wrench estimations.

This paper is structured as follows. In Sec. II the planar quadrotor dynamic equations are presented. In Sec. III, the controller is derived based on the dynamics of the planar quadrotor. Sec. IV presents the estimation process of the external actions and state. On Sec. V the results of the proposed strategy are shown and finally on Sec. VI some conclusions are drawn.

II. Planar quadrotor

In Fig. 1 is represented a simplified version of an original quadrotor in free flight with its motion restricted to the vertical plane.

The planar quadrotor pose is represented by the concatenation of its position and orientation DoFs as $\boldsymbol{p} = \begin{pmatrix} x & y & \theta \end{pmatrix}^\intercal$. Its massic properties are mass, m, and inertia, I, w.r.t. the normal plane of the picture and the origin in the mass center. They will be taken as known constants. In the following, gravity will be assumed constant in magnitude and direction.

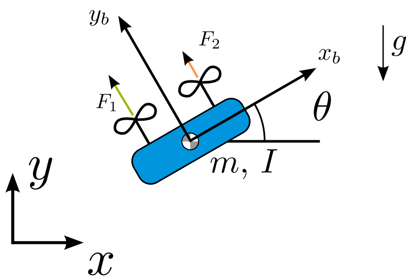

Fig. 1. Planar quadrotor.

The system is actuated with the forces created by the spinning propellers. It is usually assumed that the forces are a direct input of the system. This is a simplistic approach followed here too and commonly accepted based on the idea that sufficient fast external hardware controllers exist to control the motors.

A. Planar quadrotor dynamics

The dynamics of the quadrotor can be easily derived invoking Newton's laws. In the inertial frame given by the

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[1]http://goo.gl/pd4853

[2]AIRobots: Collaborative project ICT-248669
ARCAS: Collaborative project ICT-287617
SHERPA: Integrated Project IP-600958

origin and unitary vectors pointing in the x and y directions, the equations of motion are given by

$$\begin{aligned} -\left(F_1+F_2\right)\sin(\theta) &= m\ddot{x} \\ \left(F_1+F_2\right)\cos(\theta)-mg &= m\ddot{y} \\ \left(F_2 l - F_1 l\right) &= I\ddot{\theta}. \end{aligned} \tag{1}$$

Eq. (1) represents the mathematical model of motion of the planar quadrotor. Note that

1) The system is non-linear and coupled due to the trigonometric terms.
2) The system is under-actuated since only two actuators are present to control the six-dimensional state given by $(x,\, y,\, \theta,\, \dot{x},\, \dot{y},\, \dot{\theta})$.

B. Onboard sensors

The majority of papers dealing with external action estimation use either force sensors at particular locations were the interaction is forced to take place; or external motion capture systems on indoors or the equivalent GPS or D-GPS in outdoors. Both solutions are expensive or not completely reliable in addition of being extra systems not needed if the aircraft is going to be operated manually.

Since standard cheap inertial units provide good approximations of linear and angular velocities in addition to attitude estimations we assume here that measurements of the variables θ, $\dot{x}$, $\dot{y}$ and $\dot{\theta}$ are available at a constant rate of $100\,\mathrm{Hz}$ subject to additive zero mean measurement noise represented by the covariance matrix

$$R = diag\left(0.01,\, 0.05,\, 0.05,\, 0.05\right). \tag{2}$$

III. Non-linear impedance controller

The dynamics of the planar quadrotor presented in Eq. (1) can be seen as the interconnection of two subsystems, Σ_{11} and Σ_{12}, which describe respectively the translational motion and the rotational motion dynamics of the system. The interconnection is given by the dependency of the first subsystem with the vehicle angle as depicted in Fig. 2 and explicitly given by

$$\Sigma_1 = \begin{cases} \left. m\ddot{\boldsymbol{\xi}} = f\begin{pmatrix} -\sin(\theta) \\ \cos(\theta)\end{pmatrix} - \begin{pmatrix} 0 \\ mg \end{pmatrix} \right\} \Sigma_{11} \\ \left. I\ddot{\theta} = \tau \right\} \Sigma_{12} \end{cases}, \tag{3}$$

being $\boldsymbol{\xi}$ a vector representing the translational degrees of freedom, $\boldsymbol{\xi} = \left(x, y\right)^{\mathsf{T}}$, and f and τ linear combinations of the individual actuator forces representing the total force and the generated torque

$$\begin{pmatrix} f \\ \tau \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -l & l \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}. \tag{4}$$

Given the under-actuation of the quadrotor, two actuators and six DoFs, not all the state configurations are possible. As an example any stationary point, i.e. velocity zero; implies also $\theta = 0$. Under this point of view it can be understood that a controller can be created to stabilize the system around a trajectory comprising a subset of the states, while the other ones will take the necessary values to accomplish the commanded trajectory.

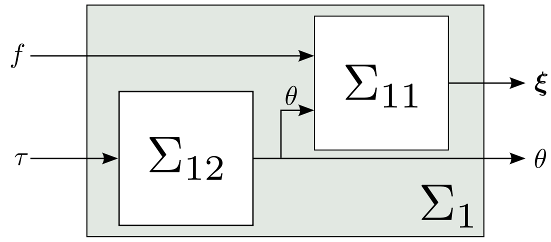

Fig. 2. Hierarchical representation of the quadrotor dynamic system

Let $\dot{\boldsymbol{\xi}}_r$ and $\ddot{\boldsymbol{\xi}}_r$ be known, i.e. provided by a planner.

Let $\boldsymbol{\mu}$ be a virtual acceleration vector (to be defined later), representing the desired acceleration for the planar quadrotor. We can use the dynamic equation of the subsystem Σ_{11} to solve the input f and the desired attitude angle θ_d that produce $\boldsymbol{\mu}$,

$$m\boldsymbol{\mu} = f\begin{pmatrix} -\sin(\theta_d) \\ \cos(\theta_d) \end{pmatrix} - \begin{pmatrix} 0 \\ mg \end{pmatrix}. \tag{5}$$

Taking into account that

$$f = m\left\| \boldsymbol{\mu} + \begin{pmatrix} 0 \\ g \end{pmatrix} \right\|, \tag{6}$$

and dividing the lateral motion equation by the vertical one

$$\tan(\theta_d) = -\frac{\mu_1}{\mu_2 + g}. \tag{7}$$

At this point, a controller can be used to drive $\theta \to \theta_d$ by knowing the linear dynamic subsystem Σ_{12} and using its input τ.

Suppose that an appropriately tuned second order filter (with natural frequency ω_n and damping factor χ) is able to deliver good approximations to the derivative of θ_d, named $\dot{\theta}_d$ and $\ddot{\theta}_d$. Defining the errors $\ddot{e}_\theta = \ddot{\theta} - \ddot{\theta}_d$, $\dot{e}_\theta = \dot{\theta} - \dot{\theta}_d$ and $e_\theta = \theta - \theta_d$ it can be seen that the choice of the control action

$$\tau = I\ddot{\theta}_d - D_\theta \dot{e}_\theta - K_\theta e_\theta, \tag{8}$$

with positive scalar constants $D_\theta > 0$ and $K_\theta > 0$, converts the dynamics of the Σ_{12} subsystem into

$$I\ddot{e}_\theta + D_\theta \dot{e}_\theta + K_\theta e_\theta = 0. \tag{9}$$

By similarity with a mechanical mass-spring-damper dynamic system it must be assumed that the error system is globally asymptotically stable and then $e_\theta \to 0$ as $t \to \infty$.

Having guaranteed the convergence of $\theta \to \theta_d$, it is needed also to ensure the convergence of $\boldsymbol{\xi} \to \boldsymbol{\xi}_r$. With this purpose, let

$$\boldsymbol{\mu} = \ddot{\boldsymbol{\xi}}_r - \frac{\mathbf{D}_\xi}{m}\dot{\boldsymbol{e}}_\xi - \frac{\mathbf{K}_\xi}{m}\boldsymbol{e}_\xi, \tag{10}$$

being $\mathbf{D}_\xi > 0$ and $\mathbf{K}_\xi > 0$ positive definite matrix, $\dot{\boldsymbol{e}}_\xi = \dot{\boldsymbol{\xi}} - \dot{\boldsymbol{\xi}}_r$ and $\boldsymbol{e}_\xi = \int \dot{\boldsymbol{e}}_\xi \mathrm{d}t$. Let, in addition, $\ddot{\boldsymbol{e}}_\xi = \ddot{\boldsymbol{\xi}} - \ddot{\boldsymbol{\xi}}_r$.

Subtracting Eq. (5) from the dynamic equation for the subsystem Σ_{11}, the translational error dynamics can be inferred as

$$m\ddot{\boldsymbol{e}}_\xi + \mathbf{D}_\xi \dot{\boldsymbol{e}}_\xi + \mathbf{K}_\xi \boldsymbol{e}_\xi = f\begin{pmatrix} \sin(\theta_d) - \sin(\theta) \\ \cos(\theta) - \cos(\theta_d) \end{pmatrix}. \tag{11}$$

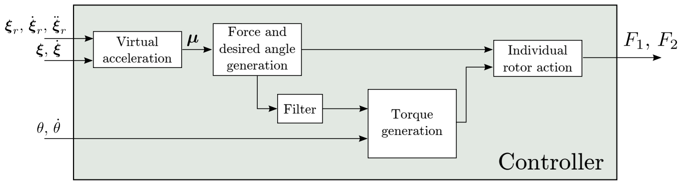

Fig. 3. Hierarchical control scheme.

Again the resultant dynamics are similar to a mass-spring-damper system, now with a forcing action. In this case, it can be ensured that if the RHS of Eq. (11), named $\boldsymbol{\delta}$ hereafter, is bounded then the error e_ξ will also be bounded by the positiveness of m, $\mathbf{D}_\xi$ and $\mathbf{K}_\xi$.

However, if the gains D_θ and K_θ are chosen to force a sufficient fast convergence of the attitude of the quadrotor to the desired angle, then it can be assumed that $\boldsymbol{\delta}$ will vanish and consequently the translational errors will converge to the origin.

Note that the derivation of the controller here given suggests a controller scheme like the presented in Fig. 3.

A. Controller modification

Up to this point, the derived controller is only useful in the case that no external disturbances in form of linear forces or torques acts over the quadrotor. However when external actions are present, the equations of motion presented in Eq. (1) become

$$\begin{aligned} -\left(F_1+F_2\right)\sin(\theta)+f_{ext_x} &= m\ddot{x} \\ \left(F_1+F_2\right)\cos(\theta)-mg+f_{ext_y} &= m\ddot{y} \\ \left(F_2 l - F_1 l\right)+\tau_{ext} &= I\ddot{\theta} \end{aligned} \tag{12}$$

If it is assumed that an estimation of the linear forces $\boldsymbol{f}_{ext} = \left(f_{ext_x},\, f_{ext_y}\right)$ and torque τ_{ext}, given by $\hat{\boldsymbol{f}}_{ext}$ and $\hat{\tau}_{ext}$ respectively are always available, then the controller outputs must be modified accordingly incorporating the estimation

$$f = m\left\|\boldsymbol{\mu}+\begin{pmatrix}0\\ g\end{pmatrix}-\frac{1}{m}\hat{\boldsymbol{f}}_{ext}\right\|, \tag{13}$$

$$\tan(\theta_d) = \frac{\hat{f}_{ext_x}-m\mu_1}{m\left(\mu_2+g\right)-\hat{f}_{ext_y}}, \tag{14}$$

and

$$\tau = I\ddot{\theta}_d - D_\theta\dot{e}_\theta - K_\theta e_\theta - \hat{\tau}_{ext}. \tag{15}$$

With this choice, the error's dynamics become

$$\begin{aligned} I\ddot{e}_\theta + D_\theta\dot{e}_\theta + K_\theta e_\theta &= \bar{\tau}_{ext} \\ m\ddot{\boldsymbol{e}}_\xi + \mathbf{D}_\xi\dot{\boldsymbol{e}}_\xi + \mathbf{K}_\xi\boldsymbol{e}_\xi &= \boldsymbol{\delta} + \bar{\boldsymbol{f}}_{ext}, \end{aligned} \tag{16}$$

being $\bar{\tau}_{ext} = \tau_{ext} - \hat{\tau}_{ext}$ and $\bar{\boldsymbol{f}}_{ext} = \boldsymbol{f}_{ext} - \hat{\boldsymbol{f}}_{ext}$. Again, given the positiveness of the coefficients the errors e_θ and $\boldsymbol{e}_\xi$ are bounded as long as $\bar{\tau}_{ext}$ and $\bar{\boldsymbol{f}}_{ext}$ are also bounded, and will tend to the origin whenever $\boldsymbol{\delta}$, $\bar{\tau}_{ext}$ and $\bar{\boldsymbol{f}}_{ext}$ vanish.

IV. Extended Kalman Filter for external force and state estimation

In order to feed the control strategy above derived, states, external forces and torques must be estimated. Since the equations of motion are non-linear, standard linear estimators as Luenberg observers or linear Kalman filters are not implementable. The EKF is a suboptimal non-linear filter that at every time step uses a linearization of the process model to provide estimations or filtered versions of measured variables based on the reconciliation between models and measures.

A. Process and measurement models

In this particular case, we will apply the EKF to estimate an augmented state vector given by $\boldsymbol{x} = (\theta,\, \dot{x},\, \dot{y},\, \dot{\theta},\, \boldsymbol{f}_{ext}, \tau_{ext})$. Since no a priori information is available for the dynamics of forces and moments, it will be assumed that they are Gaussian processes purely driven by noise, that is

$$\begin{pmatrix}\dot{\boldsymbol{f}}_{ext}\\ \dot{\tau}_{ext}\end{pmatrix} = \mathcal{N}\left(\mathbf{0},\, Q_E\right), \tag{17}$$

and by consequence the nominal dynamics of the new state vector become

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x},\boldsymbol{u}) = \begin{pmatrix} \dot{\theta} \\ \frac{1}{m}\left[-\left(F_1+F_2\right)\sin(\theta)+f_{ext_x}\right] \\ \frac{1}{m}\left[\left(F_1+F_2\right)\cos(\theta)-mg+f_{ext_y}\right] \\ \frac{1}{I}\left[\left(F_2+F_1\right)l-\tau_{ext}\right] \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{18}$$

In order to implement the EKF a discretized version of the previous dynamics are derived by using a simple Euler difference scheme

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + f(\boldsymbol{x}_k). \tag{19}$$

The measurement model relates the measures $\boldsymbol{z}_k$ with the states $\boldsymbol{x}_k$, and in this case it is

$$\boldsymbol{z}_k = \begin{pmatrix}\mathbf{I}_4 & \mathbf{0}_{4\times 3}\end{pmatrix}\boldsymbol{x}_k. \tag{20}$$

B. EKF procedure

The filter allows to produce discrete estimations of $\boldsymbol{x}$, as $\hat{\boldsymbol{x}}$ and the covariance of its difference $\hat{P}$, depending on the current estimations, the current input and the process and measure covariances, based on a two step procedure detailed in Algorithm (1)

Kalman filters model the envelope of the process error by means of the covariance of the state represented here by Q. In this case, it will be assumed that all the process uncertainty in the extended model comes from the non-perfect estimation of the external wrench. Therefore $Q = blkdiag\left(\mathbf{0}_{6\times 6},\, Q_E\right)$, where Q_E, the covariance of the wrench dynamics presented in Eq. (17), acts as tuning parameter that should be adjusted to correctly approximate the model errors.

Data: $\hat{\boldsymbol{x}}_k^-$, P_k^-, $\boldsymbol{u}$, $\boldsymbol{z}_k$, Q, R
Result: $\hat{\boldsymbol{x}}_k^+$, $\hat{\boldsymbol{x}}_{k+1}^+$, P_{k+1}^-

1) Estimate the sensor output $\hat{\boldsymbol{z}}$ from the measurement model;
2) Linearize measurement model on $\boldsymbol{x} = \hat{\boldsymbol{x}}_k^-$ and $\boldsymbol{u} \to C$;
3) Use $\mathbf{C}$ to calculate the suboptimal gain $\mathbf{K} = P_k^- C^\intercal (C P_k^- C^\intercal + R)^{-1}$;
4) Update the covariance of the estimation error $P_k^+ = (\mathbf{I} - KC)P_k^-$;
5) Update the state $\boldsymbol{x}_k^+ = \boldsymbol{x}_k^- + K(\boldsymbol{z}_k - \hat{\boldsymbol{z}})$;
6) Predict $\boldsymbol{x}_{k+1}^-$ from $\boldsymbol{x}_k^+$ and the process model ;
7) Linearize the process model on $\boldsymbol{x} = \hat{\boldsymbol{x}}_k^+ \to A$;
8) Predict the covariance at the next time step $P_{k+1}^- = AP_k^- A^\intercal + Q$

Algorithm 1: EKF algorithm

V. Results

In this section results of the derived controller are presented. In order to show the performance of the selected strategy we present three scenarios where different external forces acts over the system. The first two, does not make use of the estimator while the third does.

For all of them velocity and acceleration references are generated as

$$\begin{aligned} \dot{\boldsymbol{\xi}}_r(t) &= R\omega\left(-\sin(\omega t)\,,\cos(\omega t)\right)^\intercal \\ \dot{\boldsymbol{\xi}}_r(t) &= -R\omega^2\left(\cos(\omega t)\,,\sin(\omega t)\right)^\intercal \end{aligned} \tag{21}$$

with

$$R = 3\,\mathrm{m} \quad \omega = 0.5\,\mathrm{s}^{-1}.$$

The gains of the controllers has been kept constant in all the simulations and are presented along with the filter parameters in Table I

Parameter	Value
$\mathbf{K}_\xi$	$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$
$\mathbf{D}_\xi$	$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$
K_θ	$2 \cdot 10^{-2}$
D_θ	$4 \cdot 10^{-2}$
ω_n	20
χ	0.7
Q_E	$diag\left(9, 9, 4.8 \cdot 10^{-2}\right) \cdot 10^{-4}$
P_0^-	$diag\,(2, 2, 2, 1, 1, 1, 0, 0, 0)$

TABLE I
Controller and filter parameters for the controller and estimator of the planar quadrotor.

Fig. 4 shows the results of the first scenario, where a constant force and torque act on the body axis (this force is seen as constant in a reference frame that follows the planar quadrotor movement) with magnitudes $\boldsymbol{f}_{ext_b} = (0.3,\, 0.3)\,\mathrm{N}$ and $\tau_{ext} = 0.035\,\mathrm{Nm}$. As it can be seen, the controller without the estimator is able to cope with the deviation on the velocity trajectory that the proposed external disturbances induce. However if the external forces are increased, for example $\boldsymbol{f}_{ext_b} = (0.4,\, 0.4)\,\mathrm{N}$ and $\tau_{ext} = 0.05\,\mathrm{Nm}$, the closed loop system becomes unstable.

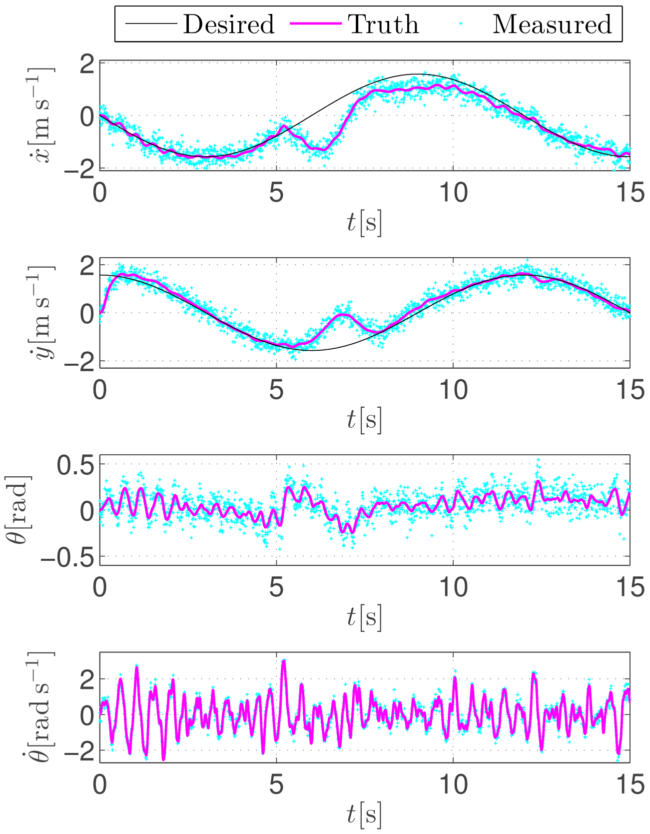

Fig. 4. Closed loop performance without wrench and state estimations.

Fig. 5 shows the result of this last scenario with the estimation of the external actions and the improved state estimations as feedback. In this case the performance of the tracking increases and the system remains stable. Fig. 6 shows the estimated forces against the real ones.

VI. Conclusions

In this paper we have presented a non-linear impedance controller with and EKF for state and external forces and torque estimation, for controlling a quadrotor-like platform in presence of external disturbances. This work is a starting point to understand how to estimate unmeasurable disturbances and how to incorporate these estimations in the control loop for the case of the under-actuated and non-linear quadrotor dynamics.

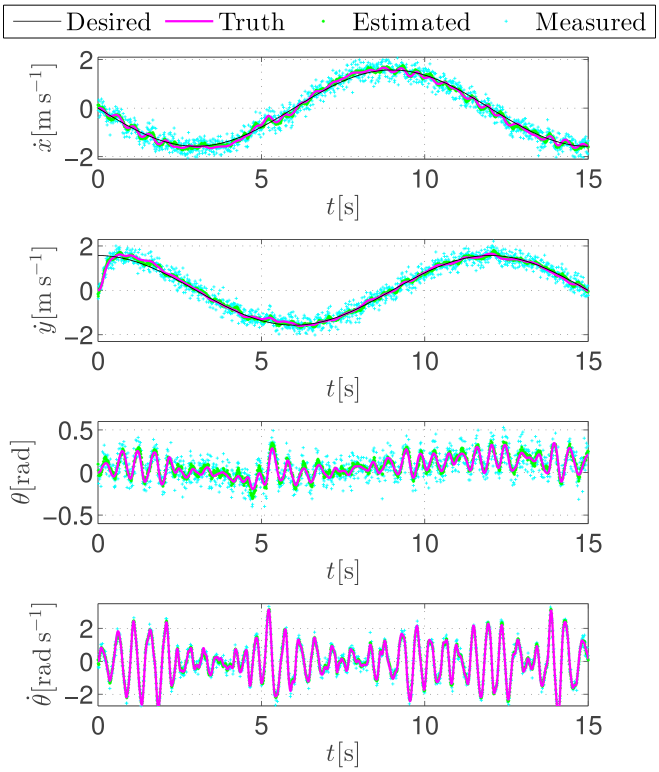

Fig. 5. Closed loop performance with wrench and state estimations.

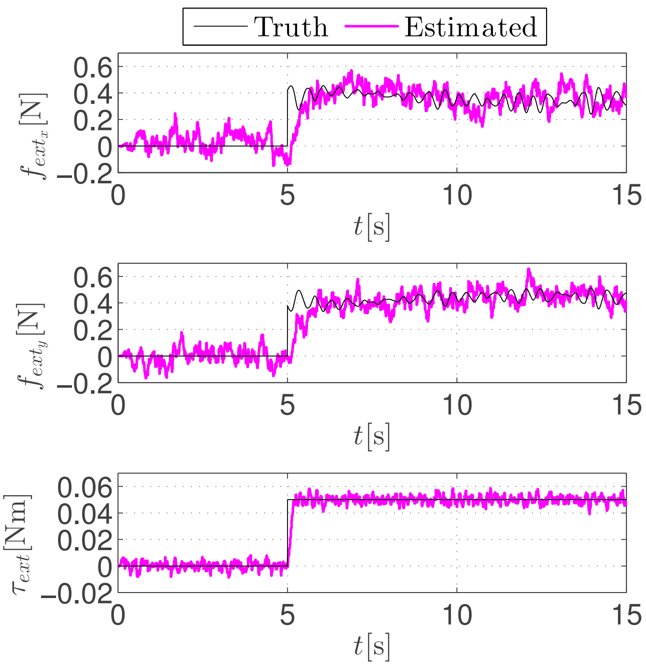

Fig. 6. Estimated external forces against real ones on the fixed reference frame.

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